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Simplified Machine Diagnosis Techniques Using n-th Moment of Absolute Deterioration Factor

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Abstract. Among many dimensional and dimensionless amplitude parameters, Kurtosis (4-th normalized moment of probability density function) is generally regarded as a sensitive good parameter for machine diagnosis. However, higher order moment may be supposed to be much more sensitive. Bicoherence is an absolute deterioration factor whose range is 1 to 0. The theoretical value of n-th moment divided by n-th moment calculated by measured data would behave in the same way. We propose a simplified calculation method for an absolute index of n-th moment and name this as simplified absolute index of n-th moment. Three cases in which the rolling elements number is nine, twelve and sixteen are examined and compared. Some favorable results are obtained.

Keywords: impact vibration, probability density function, kurtosis, n-th moment, rolling element

1. INTRODUCTION

In mass production firms such as steel making that have big equipments, sudden stops of production processes by machine failure cause severe damages such as shortage of materials to the later processes, delays to the due date and the increasing idling time.

To prevent these troubles, machine diagnosis techniques play important roles. So far, Time Based Maintenance (TBM) technique has constituted the main stream of the machine maintenance, which makes checks for maintenance at previously fixed time. But it has a weak point that it makes checks at scheduled time without taking into account whether the parts are still keeping good conditions or not. On the other hand, Condition Based Maintenance (CBM) makes maintenance checks by watching the condition of machines. Therefore, if the parts are still keeping good condition beyond its expected life, the cost of maintenance may be saved because machines can be used longer than planned. Therefore the use of CBM has become dominant. The latter one needs less cost of parts, less cost of maintenance and leads to lower failure ratio.

However, it is mandatory to catch a symptom of the failure as soon as possible of a transition from TBM to CBM is to be made. Many methods are developed and examined focusing on this subject. In this paper, we propose a method for the early detection of the failure on rotating machines which is the most common theme in machine failure detection field.

So far, many signal processing methods for machine diagnosis have been proposed (Bolleter, 1998; Hoffner, 1991). As for sensitive parameters, Kurtosis, Bicoherence, Impact Deterioration Factor (ID Factor) were examined (Yamazaki, 1977; Maekawa et al.1997; Shao et al.2001; Song et al.1998; Takeyasu, 1989). In this paper we focus our attention on the index parameters of vibration.

Kurtosis is one of the sophisticated inspection parameters which calculates normalized 4-th moment of Probability Density Function (PDF). In the industry, there are cases where quick reactions are required on watching the waveform at the machine site.

In this paper, we consider the case such that impact vibration occurs on the gear when the failure arises. Higher moments would be more sensitive compared with 4-th moment. Kurtosis value is 3.0 under normal condition and when failure increases, the value grows big. Therefore, it is a relative index. On the other hand, Bicoherence is an absolute index which is close to 1.0 under normal condition and tends to be 0 when failure increases.

In this paper, we deal with the generalized n-th moment. When theoretical value of n-th moment is divided by calculated value of n-th moment, it would behave as an absolute index. New index shows that it is 1.0 under normal condition and tends to be 0 when failure increases.

In this paper, We introduce a simplified calculation method to this new index and name this as a simplified absolute index of n-th moment. Three cases in which the rolling elements number is nine, twelve and sixteen are examined and compared.

Trying several n, we search n which shows the most similar effect to the behavior of Bicoherence. This simplified method enables us to calculate the new index even on a pocketsize calculator and enable us to install it in microcomputer chips. We survey each index of deterioration in section 2. Simplified absolute index of n-th moment is proposed in section 3. In section 4, numerical examples are presented which are followed by the remarks of section 5. Section 6 is a summary.

2. FACTORS FOR VIBRATION CALCULATION

In cyclic movements such as those of bearings and gears, the vibration grows larger whenever the deterioration becomes bigger. Also, it is well known that the vibration grows large when the setting equipment to the ground is unsuitable (Yamazaki, 1977). Let the vibration signal be presented by the function of time $x(t)$. And also assume that it is a stationary time series with mean $\bar{x}$. Denote the probability density function of these time series as $p(x)$. Indices for vibration amplitude are as follows. Here we especially suppose that $\bar{x} = 0$ for the simplicity of description.

$$X_{root} = \left[\int_{-\infty}^{\infty} |x|^{\frac{1}{2}} p(x)dx\right]^2 \tag{1}$$

$$X_{rms} = \left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{\frac{1}{2}} \tag{2}$$

$$X_{abs} = \int_{-\infty}^{\infty} |x| p(x)dx \tag{3}$$

$$X_{peak} = \lim_{n\to\infty}\left[\int_{-\infty}^{\infty} x^n p(x)dx\right]^{\frac{1}{n}} \tag{4}$$

These are dimensional indices which are not normalized. They differ by machine sizes or rotation frequencies. Therefore, normalized dimensionless indices are required.

There are four main categories for this purpose.

A. Normalized root mean square value
B. Normalized peak value
C. Normalized moment
D. Normalized correlation among frequency domain

A. Normalized root mean square value

a. Shape Factor : *SF*

$$SF = \frac{X_{rms}}{\bar{X}_{abs}} \tag{5}$$

($\bar{X}_{abs}$: mean of the absolute value of vibration)

B. Normalized peak value

b. Crest Factor : *CrF*

$$CrF = \frac{X_{peak}}{X_{rms}} \tag{6}$$

(X_{peak} : peak value of vibration)

c. Clearance Factor : *ClF*

$$ClF = \frac{X_{peak}}{X_{root}} \tag{7}$$

d. Impulse Factor : *IF*

$$IF = \frac{X_{peak}}{\bar{X}_{abs}} \tag{8}$$

e. Impact Deterioration Factor : ID Factor / *ID*

This is proposed in Maekawa *et al*. (1997).

$$ID = \frac{X_{peak}}{X_c} \tag{9}$$

(X_c : vibration amplitude where the curvature of PDF becomes maximum)

C. Normalized moment

f. Skewness : *SK*

$$SK = \frac{\int_{-\infty}^{\infty} x^3 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{\frac{3}{2}}} \tag{10}$$

g. Kurtosis : *KT*

$$KT = \frac{\int_{-\infty}^{\infty} x^4 p(x)dx}{\left[\int_{-\infty}^{\infty} x^2 p(x)dx\right]^{2}} \tag{11}$$

D. Normalized correlation in the frequency domain

h. Bicoherence

Bicoherence shows the relationship between two frequencies and is expressed as

$$Bic_{,xxx}(f_1, f_2) = \frac{B_{xxx}(f_1, f_2)}{\sqrt{S_{xx}(f_1) \cdot S_{xx}(f_2) \cdot S_{xx}(f_1+f_2)}} \tag{12}$$

Here

$$B_{xxx}(f_1, f_2) = \frac{X_T(f_1) \cdot X_T(f_2) \cdot X_T^*(f_1+f_2)}{T^{\frac{3}{2}}} \tag{13}$$

means Bispectrum and

$$X_T(t) = \begin{cases} x(t) & (0 < t < T) \\ 0 & (else) \end{cases}$$

T : Basic Frequency Interval

$$X_T(f) = \int_{-\infty}^{\infty} X_T(t) e^{-j2\pi ft} dt \tag{14}$$

$$S_{xx}(f) = \frac{1}{T} X_T(f) X_T^*(f) \tag{15}$$

Range of Bicoherence satisfies

$$0 \leq Bic_{,xxx}(f_1, f_2) \leq 1 \tag{16}$$

When there exists a significant relationship between frequencies f_1 and f_2, Bicoherence is near 1.Otherwise,the value of Bicoherence comes close to 0.

These indices are generally used in combination and machine condition is judged totally. Among them, Kurtosis is known to be one of the superior indices (Noda, 1987) and numerous researches have been conducted on Kurtosis (Maekawa *et al*.,1997; Shao *et al*.,2001; Song *et al*.,1998).

Judging from the experiment we made in the past, we may conclude that Bicoherence is also a sensitive good index (Takeyasu, 1987, 1989).

In Maekawa *et al*.(1997), ID Factor is proposed as a good index. In this paper, focusing on the indices of vibration amplitude, we introduce a simplified calculation method for absolute index of n-th moment and show that simplified absolute index of n-th moment is a sensitive good index for machine diagnosis.

3. SIMPLIFIED ABSOLUTE INDEX OF N-TH MOMENT

3.1 Absolute index of n-th moment

Mean value $\bar{x}$ of *x (t)* is calculated as

$$\bar{x} = \int_{-\infty}^{\infty} xp(x) dx$$

Discrete time series are stated as follows.

$$x_k = x(k\Delta t) \quad (k = 1, 2, \cdots)$$

Where Δt is a sampling time interval. $\bar{x}$ is stated as follows under discrete time series.

$$\bar{x} = \lim_{M \to \infty} \frac{1}{M} \sum_{i=1}^{M} x_i$$

Under the following Gaussian distribution

$$\phi(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\bar{x}}{\sigma})^2} \tag{17}$$

its moment is described as follows which is well known(Hino,1977)

$$\overline{x^{(2n-1)}} = 0 \tag{18}$$

$$\overline{x^{(2n)}} = \prod_{k=1}^{n} (2k-1)\sigma^{2n} \tag{19}$$

If we divide Eq(19) by σ^{2^n} , we can obtain normalized moment.

In general, normalized n-th moment is stated as follows.

$$Q(n)=\frac{\int_{-\infty}^{\infty}(x-\bar{x})^n p(x)dx}{\left[\int_{-\infty}^{\infty}(x-\bar{x})^2 p(x)dx\right]^{\frac{n}{2}}} \tag{20}$$

In discrete time system, it is described as

$$Q(n)=\lim_{M\to\infty}\frac{\frac{1}{M}\sum_{i=1}^{M}(x_i-\bar{x})^n}{\left\{\frac{1}{M}\sum_{i=1}^{M}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}} \tag{21}$$

We describe Q (n) as Q_N (n) if it is calculated by using N amount of data.

$$Q_N(n)=\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^n}{\left\{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}} \tag{22}$$

Absolute index of n-th moment is described as follows.

$$Z_N(n)=\frac{\prod_{k=1}^{n/2}(2k-1)}{\frac{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^n}{\left\{\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^2\right\}^{\frac{n}{2}}}} \tag{23}$$

Under the normal condition , Z_N (n) $\to 1$ $(N\to\infty)$,
and if failure becomes larger, Z_N (n) $\to 0$.

3.2 Simplified Absolute index of n-th moment

When the number of failures on bearings or gears arise, the peak values arise cyclically. In the early stage of the defect, this peak signal usually appears clearly. Generally, defects will injure other bearings or gears by contacting the inner covering surface as time passes.

Assume that we get N amount of data and then newly get L amount of data. Assume that mean, variance and moment are same with 1~N data and N+1~N+L data except for the case where a special peak signals arises.

Let mean ,variance and n-th moment calculated by using 1~N data state as

$$\bar{x}_N\ ,\ \sigma_N^2\ ,\ M_N(n)$$

And as for N+1~N+L,let them state as

$$\bar{x}_{N/l}\ ,\ \sigma_{N/l}^2\ ,\ M_{N/l}(n)$$

Where

$$M_N(n)=\frac{1}{N}\sum_{i=1}^{N}(x_i-\bar{x})^n \tag{24}$$

$$M_{N/l}(n)=\frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x})^n \tag{25}$$

Therefore, Eq(22)is stated as

$$Q_N(n) = \frac{M_N(n)}{\sigma_N^n} \tag{26}$$

Assume that the peak signal which has S times impact from normal signals arises in each *m* times samplings. As for determining the sampling interval, the sampling theorem which is well known can be used (Tokumaru *et al.*, 1982). But in this paper, we do not pay much attention on this point in order to focus on the proposed theme.
Let $\sigma^2_{N/l}$ and $M_{N/l}$ of this case, of $N+1 \sim N+l$ be $\bar{\sigma}^2_{N/l}$, $\bar{M}_{N/l}$, then we get

$$\begin{aligned} \bar{\sigma}^2_{N/l} &= \frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x})^2 \\ &= \frac{l-\frac{l}{m}}{l}\sigma_N^2 + \frac{\frac{l}{m}}{l}S^2\sigma_N^2 \\ &= \sigma_N^2\left(1+\frac{S^2-1}{m}\right) \end{aligned} \tag{27}$$

$$\begin{aligned} \bar{M}_{N/l}(n) &= \frac{1}{l}\sum_{i=N+1}^{N+l}(x_i-\bar{x})^n \\ &= \frac{l-\frac{l}{m}}{l}M_{N/l}(n) + \frac{\frac{l}{m}}{l}S^n M_{N/l} \\ &= \left(1+\frac{S^n-1}{m}\right)M_{N/l}(n) \end{aligned} \tag{28}$$

From these equations, we obtain $\bar{Q}_{N+l}$ (n)
as Q_{N+l} (n) of the above case

$$\begin{aligned} \bar{Q}_{N+l}(n) &= \frac{\frac{N}{N+l}M_N(n)+\frac{l}{N+l}\left(1+\frac{S^n-1}{m}\right)M_N(n)}{\left\{\frac{N}{N+l}\sigma_N^2+\frac{l}{N+l}\sigma_N^2\left(1+\frac{S^2-1}{m}\right)\right\}^{\frac{n}{2}}} \\ &= \frac{1+\frac{l}{N+l}\cdot\frac{S^n-1}{m}}{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^{\frac{n}{2}}}\cdot\frac{M_N(n)}{\sigma_N^2} \\ &= \frac{1+\frac{l}{N+l}\cdot\frac{S^n-1}{m}}{\left(1+\frac{l}{N+l}\cdot\frac{S^2-1}{m}\right)^{\frac{n}{2}}}Q_N(n) \end{aligned} \tag{29}$$

While $Q_N(n)$ is Kurtosis when n=4 ,

$Q_N(4)=KT$

We assume that time series are stationary as is stated before in 2. Therefore, even if sample pass may differ, mean and variance are naturally supposed to be the same when the signal is obtained from the same data occurrence point of the same machine.
We consider such case when the impact vibration occurs. Except for the impact vibration, other signals are assumed to be stationary and have the same means and variances. Under this assumption, we can derive the simplified calculation method for machine diagnosis which is a very practical one.
From the above equation, we obtain $\overline{KT}_{N+l}$ in the following way.

$$\overline{KT_{N+l}} = \frac{1+\dfrac{l}{N+l}\cdot\dfrac{S^4-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^2}\times 3.0 \tag{30}$$

Consequently, we obtain $\bar{Z}_{N+l}$ (n) as of Eq(23) as

$$\bar{Z}_{N+l}(n) = \frac{\prod_{k=1}^{n/2}(2k-1)}{\bar{Q}_{N+l}(n)}$$

$$= \frac{\prod_{k=1}^{n/2}(2k-1)}{\dfrac{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}}Q_N(n)} \tag{31}$$

Under the normal condition,

$$Q_N(n) \simeq \prod_{k=1}^{n/2}(2k-1) \tag{32}$$

Therefore, we get

$$\bar{Z}_{N+l}(n) \simeq \frac{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}}{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}} \tag{33}$$

Here we introduce the following number. Each index is compared with the normal index as follows.

$$Fa = \frac{P_{abn}}{P_{nor}} \tag{34}$$

P_{nor} : Index at normal condition

P_{abn} : Index at abnormal condition

In Eq(29), Fa becomes

$$Fa(\bar{Q}_{N+l}(n)) = \frac{1+\dfrac{l}{N+l}\cdot\dfrac{S^n-1}{m}}{\left(1+\dfrac{l}{N+l}\cdot\dfrac{S^2-1}{m}\right)^{\frac{n}{2}}} \tag{35}$$

Correlation between $\overline{Z}_{N+l}$ (n) and Fa is as follows.

$$\bar{Z}_{N+l}(n) \simeq \frac{1}{Fa(\bar{Q}_{N+l}(n))} \tag{36}$$

4. NUMERICAL EXAMPLES

If the system is under normal condition, we may suppose *p(x)* becomes a normal distribution function.

Under the assumption of 3. , let m = 9,12,16, considering the cases S = 2,4,6,8 for 3., and setting $l = \frac{N}{10}$, we obtain Table 1,3,5 from the calculation of $\bar{z}_{N+l}(n)$. Next, setting $N \to 0$, $l \to N$, we obtain Table 2,4,6. Here, m is the number of rolling elements.

Under this condition , Q (n) is as follows theoretically when n=4,6,8

Q (4) = 3.0
Q (6) = 15.0
Q (8) = 105.0
< m =9>

Table 1: Transition of $\bar{z}_{N+l}(n)$, ($m = 9$, $l = \dfrac{N}{10}$)

n	S=1	2	4	6	8
4	1	0.92185	0.37083	0.13011	0.06321
6	1	0.66837	0.03604	0.00525	0.00165

Table 2: Transition of $\bar{z}_{N+l}(n)$ ($m = 9$, $N = \varepsilon(\varepsilon \to 0)$, $l = N$)

n	S=1	2	4	6	8
4	1	0.66667	0.24242	0.16496	0.14035
6	1	0.29630	0.04159	0.02254	0.01758

< m =12>

Table 3: Transition of $\bar{z}_{N+l}(n)$, ($m = 12$, $l = \dfrac{N}{10}$)

n	S=1	2	4	6	8
4	1	0.93924	0.42301	0.14806	0.06815
6	1	0.72413	0.04313	0.00571	0.00162

Table 4: Transition of $\bar{z}_{N+l}(n)$ ($m = 12$, $N = \varepsilon(\varepsilon \to 0)$, $l = N$)

n	S=1	2	4	6	8
4	1	0.69444	0.22753	0.14084	0.11413
6	1	0.31250	0.03328	0.01545	0.01118

< m =16>

Table 5: Transition of $\bar{z}_{N+l}(n)$, ($m = 16$, $l = \dfrac{N}{10}$)

n	S=1	2	4	6	8
4	1	0.95315	0.48092	0.17196	0.07599
6	1	0.77470	0.05267	0.00648	0.00168

Table 6: Transition of $\bar{z}_{N+l}(n)$ ($m = 16$, $N = \varepsilon(\varepsilon \to 0)$, $l = N$)

n	S=1	2	4	6	8
4	1	0.72782	0.22163	0.12400	0.09488
6	1	0.33915	0.02839	0.01110	0.00735

As S grows large, the value decreases rapidly and as n grows large, the value also decreases rapidly. In the case of n=6, the value is already so small, therefore there is no need to calculate the case of n=8.

Subsequently, we examine Bicoherence. We made experiment in the past (Takeyasu,1987,1989).

Summary of the experiment is as follows. Pitching defects are pressed on the gears of small testing machine.

Small defect condition: Pitching defects pressed on 1/3 gears of the total gear.

Middle defect condition: Pitching defects pressed on 2/3 gears of the total gear.

Big defect condition: Pitching defects pressed on whole gears of the total gear.

We examined several cases for the f_1 , f_2 in Eq.(12). We got best-fit result in the following case.

$$\begin{cases} f_1 : \text{peak frequency of power spectrum} \\ f_2 : 2\ f_1 \end{cases}$$

We obtained following Bicoherence values in this case (Table3).

Table3: Transition of Bicoherence value

Condition	Bicoherence
Normal	0.99
Small defect	0.38
Middle defect	0.09
Big defect	0.02

Thus, Bicoherence proved to be a very sensitive good index. Bicoherence is an absolute index of which range is 1 to 0.

Therefore it can be said that it is a universal index.

In those experiment, small defect condition is generally assumed to be S=2 and big defect condition is generally assumed to be S=6 (Maekawa,K. et al. 1997). Therefore, approximate comparison may be achieved, though the condition does not necessarily coincide.

Now, we compare this proposed simplified absolute index of n-th moment with those of Bicoherence. The proposed method is an absolute index of which range is from 1 to 0 similarly as Bicoherence. As for sensitivity, the case of n=4 is quite similar to Bicoherence, and the proposed one in the case of n=6 is much more sensitive. It is suitable for especially early stage failure detection.

It could be said that this method would be sensitive enough for the practical use. This calculation method is simple enough to be executed even on a pocketsize calculator. Compared with Bicoherence which has to be calculated by Eq.(12) ~ (15), the proposed method is by far a simple one and easy to handle on the field defection.

5. REMARKS

Here, we introduced firstly an absolute index of n-th moment and then a simplified calculation method for a simplified absolute index of n-th moment. We compared proposed method with Bicoherence.

The result of this simplified calculation method is a reasonable one compared with the results obtained so far.

The steps for the failure detection by this method are as follows.

1. Prepare a standard $\overline{Z}$ Table for each normal or abnormal level

2. Measure peak values by signal data and compare the peak ratio to the normal data

3. Calculate $\overline{Z}$ by Eq(33)

4. Judge the failure level by the score of $\overline{Z}$

Generally, it is said that machine is under small defect condition when S=2.

By precise diagnosis utilizing deterioration index such as Bicoherence or equation (33), much more detailed machine condition can be estimated.

How to set the abnormal level of the machine depends upon the level of control.

If the machine is very important and they must be maintained carefully, subtle change of deterioration index should not be neglected. They have to make maintenance at a rather small change of deterioration index. If not, maintenance may be executed at a rather big value of index (This is a case that big value of index means increasing damages).

Preparing standard Table of $\overline{Z}$ for each normal and abnormal level, we can easily judge the failure level only by taking ratio of the peak value to the normal level and calculating $\overline{Z}$ by (33). This method is simple enough to be carried out even on a pocketsize calculator and is very practical at the factory of maintenance site. This can be installed in microcomputer chips and utilized as the tool for early stage detection of the failure.

6. CONCLUSIONS

We proposed a simplified calculation method for an absolute index of n-th moment and named this as simplified absolute index of n-th moment. Three cases in which the rolling elements number was nine, twelve and sixteen were examined and compared. As S grows large, the value decreases rapidly and as n grows large, the value also decreases rapidly. As for sensitivity, the case of n=4 was quite similar to Bicoherence, and the proposed one in the case of n=6 was much more sensitive. It is suitable for especially early stage failure detection. Compared with the results obtained so far, the results of numerical examples of this paper are reasonable. Judging from these results, our method is properly considered to be effective especially for early stage failure detection. This calculation method is simple enough to be executed even on a pocketsize calculator and is very practical at the factory of maintenance site. The effectiveness of this method should be examined in various cases.

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