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Expansion of the Block Matrix to the Second Order Lag in Brand Selection

Kazuhiro TAKEYASU

ABSTRACT：Focusing on the fact that consumers are apt to buy superior brand when they are accustomed or bored to use the current brand, a new analysis method is introduced. The data set (before buying data and after buying data (for example, former buying data and current buying data)) is stated using a liner model. When this is done, the transition matrix becomes an upper triangular matrix. In this paper, equation using the transition matrix stated by the Block Matrix is expanded to the second order lag and the method is newly re-built. These are confirmed by numerical examples. An S-step forecasting model is also introduced. This approach makes it possible to identify brand position in the market and it can be utilized for building a useful and effective marketing plan.

Key Words: brand selection, matrix structure, brand position, second order lag

1. INTRODUCTION

It is often observed that consumers select upper-class brand when they buy the next time after they are bored to use the current brand. Focusing the transition matrix structure of brand selection, their activities may be analyzed. In the past, there are many researches about brand selection [1-5]. But there are few papers concerning the analysis of the transition matrix structure of brand selection. In this paper, we make analysis of the preference shift of customer brand selection and confirm them by the questionnaire investigation for automobile purchasing case. If we can identify the feature of the matrix structure of brand selection, it can be utilized for the marketing strategy.

Suppose that the former buying data and the current buying data are gathered. Also suppose that upper brand is located upper in the variable array. Then the transition matrix becomes an upper triangular matrix under the supposition that the former buying variables are used as input and the current buying variables are used as output. If the transition matrix is identified, an s-step forecasting can be executed. Generalized forecasting matrix components' equations are introduced. Planners for products need to know whether their brand is upper or lower than other products. Matrix structure makes it possible to ascertain this by calculating consumers' activities for brand selection. Thus, this proposed approach makes it possible to execute an effective marketing plan and/or establish new brand.

Quantitative analysis concerning brand selection has been conducted by Yamanaka[5], Takahashi et al.[4]. Yamanaka[5] examined purchasing process by Markov Transition Probability with the input of advertising expense. Takahashi et al.[4] made analysis by the Brand Selection Probability model using logistics distribution. Heung-Suk Hwang et al.[6] made a research concerning supplier selection using AHP. But it is not the theme we are handling now.

In Takeyasu et al. (2007,2011), matrix structure was analyzed for the case brand selection was carried out toward upper-class. In this paper, equation using transition matrix stated by the Block matrix is extended to the second order lag and the method is newly re-built. Such research as this cannot be found as long as searched. Utilizing this method, we can easily find the brand position where they are and we can establish effective marketing plan.

Hereinafter, matrix structure is clarified for the selection of brand in section 2. Block matrix structure is analyzed when brands are handled in group and an s-step forecasting is formulated in section 3. Expansion of the model to the second order lag is executed in section 4. Numerical calculation is executed in section 5. Application of this method is extended in section 6.

2. BRAND SELECTION AND ITS MATRIX STRUCTURE

Now, suppose that x is the most upper class brand, y is the second upper class brand, and z is the lowest class brand. Consumer's behavior of selecting brand might be $z \rightarrow y$, $y \rightarrow x$, $z \rightarrow x$ etc. $x \rightarrow z$ might be few.

Suppose that x is current buying variable, and x_b is previous buying variable. Shift to x is executed from x_b, y_b, or z_b.

Therefore, x is stated in the following equation. a_{ij} represents the transition probability from j-th to i-th brand.

$$x = a_{11}x_b + a_{12}y_b + a_{13}z_b$$

Similarly,

$$y = a_{22}y_b + a_{23}z_b$$

and

$$z = a_{33}z_b$$

These are re-written as follows.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix} \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} \tag{1}$$

Set

$$\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{pmatrix}, \qquad \mathbf{X_b} = \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix}$$

then, $\mathbf{X}$ is represented as follows.

$$\mathbf{X} = \mathbf{A}\mathbf{X_b} \tag{2}$$

Here,

$$\mathbf{X} \in \mathbf{R}^3, \mathbf{A} \in \mathbf{R}^{3\times3}, \mathbf{X_b} \in \mathbf{R}^3$$

$\mathbf{A}$ is an upper triangular matrix.
To examine this, generating the following data, which all consist of the data in which transition is made from a lower-class brand to an upper-class brand,

$$\mathbf{X^i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{X_b^i} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{4}$$

$$i = 1 \quad , \quad 2 \quad \cdots \quad N$$

parameter can be estimated by using least square method.
Suppose

$$\mathbf{X}^i = \mathbf{A}\mathbf{X}_\mathbf{b}^i + \boldsymbol{\varepsilon}^i \tag{5}$$

where $\varepsilon^i = \begin{pmatrix} \varepsilon_1^i \\ \varepsilon_2^i \\ \varepsilon_3^i \end{pmatrix} \qquad i = 1,2,\cdots,N$

and minimize following J

$$J = \sum_{i=1}^{N} \boldsymbol{\varepsilon}^{iT} \boldsymbol{\varepsilon}^i \rightarrow Min \tag{6}$$

$\hat{\mathbf{A}}$ which is an estimated value of $\mathbf{A}$ is obtained as follows.

$$\hat{\mathbf{A}} = \left(\sum_{i=1}^{N} \mathbf{X}^i \mathbf{X}_\mathbf{b}^{iT} \right) \left(\sum_{i=1}^{N} \mathbf{X}_\mathbf{b}^i \mathbf{X}_\mathbf{b}^{iT} \right)^{-1} \tag{7}$$

In the data group which are all consisted by the data in which transition is made from a lower-class brand to an upper-class brand, an estimated value $\hat{\mathbf{A}}$ should be an upper triangular matrix.
If following data which shift to lower brand are added only a few in equation (3) and (4),

$$\mathbf{X}^i = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{X}_\mathbf{b}^i = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$\hat{\mathbf{A}}$ would contain minute items in the lower part of triangle.

3. BLOCK MATRIX STRUCTURE IN BRAND GOURPS AND S-STEP FORECASTING

Next, we examine the case in brand groups. Matrices are composed by Block Matrix.

(1) Brand shift group — in the case of two groups

Suppose brand selection shifts from Corolla class to Mark Ⅱ class in car. In this case, it does not matter which company's car they choose. Thus, selection of cars are executed in a group and brand shift is considered to be done from group to group. Suppose brand groups at time n are as follows.

$\mathbf{X}$ consists of p varieties of goods, and $\mathbf{Y}$ consists of q varieties of goods.

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \qquad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}} \\ \mathbf{0}, & \mathbf{A_{22}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \end{pmatrix} \tag{8}$$

Here,

$$\mathbf{X_n} \in \mathbf{R}^p \ (n = 1,2,\cdots), \quad \mathbf{Y_n} \in \mathbf{R}^q \ (n = 1,2,\cdots), \quad \mathbf{A_{11}} \in \mathbf{R}^{p\times p}, \quad \mathbf{A_{12}} \in \mathbf{R}^{p\times q}, \quad \mathbf{A_{22}} \in \mathbf{R}^{q\times q}$$

Finally, we get generalized equation for s-step shift as follows.

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}^s, & \mathbf{A_{11}}^{s-1}\mathbf{A_{12}} + \sum_{k=2}^{s-1} \mathbf{A_{11}}^{s-k}\mathbf{A_{12}}\mathbf{A_{22}}^{k-1} + \mathbf{A_{12}}\mathbf{A_{22}}^{s-1} \\ \mathbf{0}, & \mathbf{A_{22}}^s \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-s}} \\ \mathbf{Y_{n-s}} \end{pmatrix} \tag{9}$$

If we replace $n-s \to n, n \to n+s$ in equation (9), we can make s-step forecast.

N.B. "," in the matrix is used in the case equations become complex and it must be separated so as not to confuse. Serial matrix equation groups are expressed in the same way.

(2) Brand shift group — in the case of three groups

Suppose brand selection is executed in the same group or to the upper group, and also suppose that brand position is $x > y > z$ (x is upper position). Then brand selection transition matrix would be expressed as:

$$\begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix} = \begin{pmatrix} \mathbf{A_{11}}, & \mathbf{A_{12}}, & \mathbf{A_{13}} \\ \mathbf{0}, & \mathbf{A_{22}}, & \mathbf{A_{23}} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}} \end{pmatrix} \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix} \tag{10}$$

Where

$$\mathbf{X_n} = \begin{pmatrix} x_1^n \\ x_2^n \\ \vdots \\ x_p^n \end{pmatrix}, \qquad \mathbf{Y_n} = \begin{pmatrix} y_1^n \\ y_2^n \\ \vdots \\ y_q^n \end{pmatrix}, \qquad \mathbf{Z_n} = \begin{pmatrix} z_1^n \\ z_2^n \\ \vdots \\ z_r^n \end{pmatrix}$$

Here,

$$\mathbf{X_n} \in \mathbf{R}^p \ (n = 1,2,\cdots), \quad \mathbf{Y_n} \in \mathbf{R}^q \ (n = 1,2,\cdots), \quad \mathbf{Z_n} \in \mathbf{R}^r \ (n = 1,2,\cdots), \quad \mathbf{A_{11}} \in R^{p\times p}, \quad \mathbf{A_{12}} \in R^{p\times q}$$

$$\mathbf{A_{13}} \in R^{p\times r}, \quad \mathbf{A_{22}} \in R^{q\times q}, \quad \mathbf{A_{23}} \in R^{q\times r}, \quad \mathbf{A_{33}} \in R^{r\times r}$$

These are re-stated as:

$$\mathbf{W_n} = \mathbf{A}\mathbf{W_{n-1}} \tag{11}$$

where,

$$\mathbf{W_n} = \begin{pmatrix} \mathbf{X_n} \\ \mathbf{Y_n} \\ \mathbf{Z_n} \end{pmatrix}, \qquad \mathbf{A} = \begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix}, \qquad \mathbf{W_{n-1}} = \begin{pmatrix} \mathbf{X_{n-1}} \\ \mathbf{Y_{n-1}} \\ \mathbf{Z_{n-1}} \end{pmatrix}$$

Hereinafter, we shift steps as is done in previous section.
In the general description, we state as:

$$\mathbf{W_n} = \mathbf{A^{(s)}}\mathbf{W_{n-s}} \tag{12}$$

Here,

$$\mathbf{A^{(s)}} = \begin{pmatrix} \mathbf{A_{11}}^{\mathbf{(s)}}, & \mathbf{A_{12}}^{\mathbf{(s)}}, & \mathbf{A_{13}}^{\mathbf{(s)}} \\ \mathbf{0}, & \mathbf{A_{22}}^{\mathbf{(s)}}, & \mathbf{A_{23}}^{\mathbf{(s)}} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A_{33}}^{\mathbf{(s)}} \end{pmatrix}, \qquad \mathbf{W_{n-s}} = \begin{pmatrix} \mathbf{X_{n-s}} \\ \mathbf{Y_{n-s}} \\ \mathbf{Z_{n-s}} \end{pmatrix}$$

From definition,

$$\mathbf{A^{(1)}} = \mathbf{A} \tag{13}$$

In the case $s = 2$, we obtain

$$\begin{aligned} \mathbf{A}^{(2)} &= \begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{A}_{11}, & \mathbf{A}_{12}, & \mathbf{A}_{13} \\ \mathbf{0}, & \mathbf{A}_{22}, & \mathbf{A}_{23} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{A}_{11}^2, & \mathbf{A}_{11}\mathbf{A}_{12} + \mathbf{A}_{12}\mathbf{A}_{22}, & \mathbf{A}_{11}\mathbf{A}_{13} + \mathbf{A}_{12}\mathbf{A}_{23} + \mathbf{A}_{13}\mathbf{A}_{33} \\ \mathbf{0}, & \mathbf{A}_{22}^2, & \mathbf{A}_{22}\mathbf{A}_{23} + \mathbf{A}_{23}\mathbf{A}_{33} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^2 \end{pmatrix} \end{aligned} \tag{14}$$

We get generalized equations for s-step shift as follows.

$$\left.\begin{aligned} \mathbf{A}_{11}^{(s)} &= \mathbf{A}_{11}^{s} \\ \mathbf{A}_{12}^{(s)} &= \mathbf{A}_{11}^{s-1}\mathbf{A}_{12} + \sum_{k=2}^{s-1}\mathbf{A}_{11}^{s-k}\mathbf{A}_{12}\mathbf{A}_{22}^{k-1} + \mathbf{A}_{12}\mathbf{A}_{22}^{s-1} \\ \mathbf{A}_{13}^{(s)} &= \mathbf{A}_{11}^{s-1}\mathbf{A}_{13} + \mathbf{A}_{11}^{s-2}\left(\sum_{k=1}^{2}\mathbf{A}_{\mathbf{1(k+1)}}\mathbf{A}_{\mathbf{(k+1)3}}\right) + \sum_{j=1}^{s-3}\left[\mathbf{A}_{11}^{S-2-j}\left\{\mathbf{A}_{12}\left(\sum_{k=1}^{j+1}\mathbf{A}_{22}^{j+1-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}\right) + \mathbf{A}_{13}\mathbf{A}_{33}^{j+1}\right\}\right] \\ \mathbf{A}_{22}^{(s)} &= \mathbf{A}_{22}^{s} \\ \mathbf{A}_{23}^{(s)} &= \sum_{K-1}^{s}\mathbf{A}_{22}^{s-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1} \\ \mathbf{A}_{33}^{(s)} &= \mathbf{A}_{33}^{s} \end{aligned}\right\} \tag{15}$$

Expressing them in the matrix form, it follows:

$$\mathbf{A}^{(S)}=\begin{pmatrix} \mathbf{A}_{11}^{s}, & \mathbf{A}_{11}^{s-1}\mathbf{A}_{12}+\sum_{k=2}^{s-1}\mathbf{A}_{11}^{s-k}\mathbf{A}_{12}\mathbf{A}_{22}^{k-1}+\mathbf{A}_{12}\mathbf{A}_{22}^{s-1}, & \mathbf{A}_{11}^{s-1}\mathbf{A}_{13}+\mathbf{A}_{11}^{s-2}\left(\sum_{k=1}^{2}\mathbf{A}_{1(k+1)}\mathbf{A}_{(K+1)3}\right)+\sum_{j=1}^{s-3}\left[\mathbf{A}_{11}^{s-2-j}\left\{\mathbf{A}_{12}\left(\sum_{k=1}^{j+1}\mathbf{A}_{22}^{j+1-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1}\right)+\mathbf{A}_{13}\mathbf{A}_{33}^{j+1}\right\}\right] \\ \mathbf{0}, & \mathbf{A}_{22}^{s}, & \sum_{k=1}^{s}\mathbf{A}_{22}^{s-k}\mathbf{A}_{23}\mathbf{A}_{33}^{k-1} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{A}_{33}^{s} \end{pmatrix} \tag{16}$$

Generalizing them to m groups, they are expressed as:

$$\begin{pmatrix}\mathbf{X}_{\mathbf{n}}^{(1)}\\ \mathbf{X}_{\mathbf{n}}^{(2)}\\ \vdots \\ \mathbf{X}_{\mathbf{n}}^{(m)}\end{pmatrix}=\begin{pmatrix}\mathbf{A}_{11} & \mathbf{A}_{12} & \cdots & \mathbf{A}_{1m}\\ \mathbf{A}_{21} & \mathbf{A}_{22} & \cdots & \mathbf{A}_{2m}\\ \vdots & \vdots & & \vdots\\ \mathbf{A}_{m1} & \mathbf{A}_{m2} & \cdots & \mathbf{A}_{mm}\end{pmatrix}\begin{pmatrix}\mathbf{X}_{\mathbf{n-1}}^{(1)}\\ \mathbf{X}_{\mathbf{n-1}}^{(2)}\\ \vdots \\ \mathbf{X}_{\mathbf{n-1}}^{(m)}\end{pmatrix} \tag{17}$$

$$\mathbf{X}_{\mathbf{n}}^{(1)}\in R^{k_1},\quad \mathbf{X}_{\mathbf{n}}^{(2)}\in R^{k_2},\ \cdots,\ \mathbf{X}_{\mathbf{n}}^{(m)}\in R^{k_m},\quad \mathbf{A}_{\mathbf{ij}}\in R^{k_i\times k_j}\ (i=1,\cdots,m)(j=1,\cdots,m)$$

4. Expansion to the second order lag

Expansion of the above stated Block Matrix model to the second order lag is performed in the following method. Here we take three groups case.

Generating Eq.(10) and Eq.(12),we state the model as follows. Here we set P=3.

$$\begin{pmatrix}\mathbf{X}_n\\ \mathbf{Y}_n\\ \mathbf{Z}_n\end{pmatrix}=\begin{pmatrix}\mathbf{A}, & \mathbf{B}, & \mathbf{C}\\ \mathbf{D}, & \mathbf{E}, & \mathbf{F}\\ \mathbf{G}, & \mathbf{H}, & \mathbf{J}\end{pmatrix}\begin{pmatrix}\mathbf{X}_{n-1}\\ \mathbf{Y}_{n-1}\\ \mathbf{Z}_{n-1}\end{pmatrix} \tag{18}$$

Where

$$\mathbf{X}_n=\begin{pmatrix}x_1^n\\ x_2^n\\ x_3^n\end{pmatrix},\ \mathbf{Y}_n=\begin{pmatrix}y_1^n\\ y_2^n\\ y_3^n\end{pmatrix},\ \mathbf{Z}_{\mathbf{n}}=\begin{pmatrix}z_1^n\\ z_2^n\\ z_3^n\end{pmatrix} \tag{19}$$

Here,

$$\mathbf{X}_n\in\mathbf{R}^3(n=1,2,\cdots),\ \mathbf{Y}_n\in\mathbf{R}^3(n=1,2,\cdots),\ \mathbf{Z}_n\in\mathbf{R}^3(n=1,2,\cdots),\ \{\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D},\mathbf{E},\mathbf{F},\mathbf{G},\mathbf{H},\mathbf{J}\}\in\mathbf{R}^{3\times 3}$$

These are re-stated as:

$$\mathbf{W}_n=\mathbf{P}\mathbf{W}_{n-1} \tag{20}$$

$$\mathbf{W}_n=\begin{pmatrix}\mathbf{X}_n\\ \mathbf{Y}_n\\ \mathbf{Z}_n\end{pmatrix} \tag{21}$$

$$\mathbf{P}=\begin{pmatrix}\mathbf{A}, & \mathbf{B}, & \mathbf{C}\\ \mathbf{D}, & \mathbf{E}, & \mathbf{F}\\ \mathbf{G}, & \mathbf{H}, & \mathbf{J}\end{pmatrix} \tag{22}$$

$$\mathbf{W}_{n-1}=\begin{pmatrix}\mathbf{X}_{n-1}\\ \mathbf{Y}_{n-1}\\ \mathbf{Z}_{n-1}\end{pmatrix} \tag{23}$$

If N amount of data exist, we can derive the following equation similarly as Eq.(5),

$$\mathbf{W}_n^i=\mathbf{P}\mathbf{W}_{n-1}^i+\boldsymbol{\varepsilon}_n^i\left(i=1,2,\cdots,N\right) \tag{24}$$

and

$$J_n=\sum_{i=1}^{N}\boldsymbol{\varepsilon}_n^{iT}\boldsymbol{\varepsilon}_n^i\rightarrow Min \tag{25}$$

$\hat{\mathbf{P}}$ which is an estimated value of $\mathbf{P}$ is obtained as follows.

$$\hat{\mathbf{P}}=\left(\sum_{i=1}^{N}\mathbf{W}_n^i\mathbf{W}_{n-1}^{iT}\right)\left(\sum_{i=1}^{N}\mathbf{W}_{n-1}^i\mathbf{W}_{n-1}^{iT}\right)^{-1} \tag{26}$$

Now, we expand Eq.(24) to the second order lag model as follows.

$$\mathbf{W}_n^i=\mathbf{P}_1\mathbf{W}_{n-1}^i+\mathbf{P}_2\mathbf{W}_{n-2}^i+\boldsymbol{\varepsilon}_n^i \tag{27}$$

Here

$$\mathbf{P}_1=\begin{pmatrix}\mathbf{A}_1, & \mathbf{B}_1, & \mathbf{C}_1\\ \mathbf{D}_1, & \mathbf{E}_1, & \mathbf{F}_1\\ \mathbf{G}_1, & \mathbf{H}_1, & \mathbf{J}_1\end{pmatrix},\mathbf{P}_2=\begin{pmatrix}\mathbf{A}_2, & \mathbf{B}_2, & \mathbf{C}_2\\ \mathbf{D}_2, & \mathbf{E}_2, & \mathbf{F}_2\\ \mathbf{G}_2, & \mathbf{H}_2, & \mathbf{J}_2\end{pmatrix} \tag{28}$$

It we set

$$\mathbf{P}=\left(\mathbf{P}_1,\mathbf{P}_2\right) \tag{29}$$

then $\hat{\mathbf{P}}$ can be estimated as follows.

$$\hat{\mathbf{P}}=\left(\sum_{i=1}^{N}\mathbf{W}_t^i\begin{pmatrix}\mathbf{W}_{t-1}^i\\ \mathbf{W}_{t-2}^i\end{pmatrix}^T\right)\left(\sum_{i=1}^{N}\begin{pmatrix}\mathbf{W}_{t-1}^i\\ \mathbf{W}_{t-2}^i\end{pmatrix}\begin{pmatrix}\mathbf{W}_{t-1}^i\\ \mathbf{W}_{t-2}^i\end{pmatrix}^T\right)^{-1} \tag{30}$$

We further develop this equation as follows.

$$\hat{\mathbf{P}}=\left(\hat{\mathbf{P}}_1,\hat{\mathbf{P}}_2\right)$$

$$=\begin{pmatrix}\mathbf{A}_1, & \mathbf{B}_1, & \mathbf{C}_1, & \mathbf{A}_2, & \mathbf{B}_2, & \mathbf{C}_2\\ \mathbf{D}_1, & \mathbf{E}_1, & \mathbf{F}_1, & \mathbf{D}_2, & \mathbf{E}_2, & \mathbf{F}_2\\ \mathbf{G}_1, & \mathbf{H}_1, & \mathbf{J}_1, & \mathbf{G}_2, & \mathbf{H}_2, & \mathbf{J}_2\end{pmatrix}$$

$$=\left(\sum_{i=1}^{N}\mathbf{W}_t^i\mathbf{W}_{t-1}^{iT},\quad \sum_{i=1}^{N}\mathbf{W}_t^i\mathbf{W}_{t-2}^{iT}\right)\begin{pmatrix}\sum_{i=1}^{N}\mathbf{W}_{t-1}^i\mathbf{W}_{t-1}^{iT}, & \sum_{i=1}^{N}\mathbf{W}_{t-1}^i\mathbf{W}_{t-2}^{iT}\\ \sum_{i=1}^{N}\mathbf{W}_{t-2}^i\mathbf{W}_{t-1}^{iT}, & \sum_{i=1}^{N}\mathbf{W}_{t-2}^i\mathbf{W}_{t-2}^{iT}\end{pmatrix}^{-1}$$

$$=\left(\sum_{i=1}^{N}\begin{pmatrix}\mathbf{x}_t^i\\ \mathbf{y}_t^i\\ \mathbf{z}_t^i\end{pmatrix}\left(\mathbf{x}_{t-1}^{iT},\mathbf{y}_{t-1}^{iT},\mathbf{z}_{t-1}^{iT}\right),\quad \sum_{i=1}^{N}\begin{pmatrix}\mathbf{x}_t^i\\ \mathbf{y}_t^i\\ \mathbf{z}_t^i\end{pmatrix}\left(\mathbf{x}_{t-2}^{iT},\mathbf{y}_{t-2}^{iT},\mathbf{z}_{t-2}^{iT}\right)\right)$$

$$
\times \begin{pmatrix} \sum_{i=1}^{N} \begin{pmatrix} \mathbf{x}_{t-1}^{i} \\ \mathbf{y}_{t-1}^{i} \\ \mathbf{z}_{t-1}^{i} \end{pmatrix} \left(\mathbf{x}_{t-1}^{iT}, \mathbf{y}_{t-1}^{iT}, \mathbf{z}_{t-1}^{iT} \right), & \sum_{i=1}^{N} \begin{pmatrix} \mathbf{x}_{t-1}^{i} \\ \mathbf{y}_{t-1}^{i} \\ \mathbf{z}_{t-1}^{i} \end{pmatrix} \left(\mathbf{x}_{t-2}^{iT}, \mathbf{y}_{t-2}^{iT}, \mathbf{z}_{t-2}^{iT} \right) \\ \sum_{i=1}^{N} \begin{pmatrix} \mathbf{x}_{t-2}^{i} \\ \mathbf{y}_{t-2}^{i} \\ \mathbf{z}_{t-2}^{i} \end{pmatrix} \left(\mathbf{x}_{t-1}^{iT}, \mathbf{y}_{t-1}^{iT}, \mathbf{z}_{t-1}^{iT} \right), & \sum_{i=1}^{N} \begin{pmatrix} \mathbf{x}_{t-2}^{i} \\ \mathbf{y}_{t-2}^{i} \\ \mathbf{z}_{t-2}^{i} \end{pmatrix} \left(\mathbf{x}_{t-2}^{iT}, \mathbf{y}_{t-2}^{iT}, \mathbf{z}_{t-2}^{iT} \right) \end{pmatrix}^{-1}
$$

$$
= \left(\begin{array}{ccc:ccc} \sum_{i=1}^{N} \mathbf{x}_{t}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t}^{i} \mathbf{z}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{y}_{t}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t}^{i} \mathbf{z}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{z}_{t}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t}^{i} \mathbf{z}_{t-2}^{iT} \end{array} \right)
$$

$$
\times \left(\begin{array}{ccc:ccc} \sum_{i=1}^{N} \mathbf{x}_{t-1}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-1}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-1}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-1}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-1}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-1}^{i} \mathbf{z}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{y}_{t-1}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-1}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-1}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-1}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-1}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-1}^{i} \mathbf{z}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{z}_{t-1}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-1}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-1}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-1}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-1}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-1}^{i} \mathbf{z}_{t-2}^{iT} \\ \hdashline \sum_{i=1}^{N} \mathbf{x}_{t-2}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-2}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-2}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-2}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-2}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{x}_{t-2}^{i} \mathbf{z}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{y}_{t-2}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-2}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-2}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-2}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-2}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{y}_{t-2}^{i} \mathbf{z}_{t-2}^{iT} \\ \sum_{i=1}^{N} \mathbf{z}_{t-2}^{i} \mathbf{x}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-2}^{i} \mathbf{y}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-2}^{i} \mathbf{z}_{t-1}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-2}^{i} \mathbf{x}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-2}^{i} \mathbf{y}_{t-2}^{iT}, & \sum_{i=1}^{N} \mathbf{z}_{t-2}^{i} \mathbf{z}_{t-2}^{iT} \end{array} \right)^{-1} \quad (31)
$$

We set this as:

$$
\hat{\mathbf{P}} = \left(\begin{array}{ccc:ccc} \mathbf{K}_1 & \mathbf{K}_2 & \mathbf{K}_3 & \mathbf{L}_1 & \mathbf{L}_2 & \mathbf{L}_3 \\ \mathbf{K}_4 & \mathbf{K}_5 & \mathbf{K}_6 & \mathbf{L}_4 & \mathbf{L}_5 & \mathbf{L}_6 \\ \mathbf{K}_7 & \mathbf{K}_8 & \mathbf{K}_9 & \mathbf{L}_7 & \mathbf{L}_8 & \mathbf{L}_9 \end{array} \right)
$$

$$
\times \left(\begin{array}{ccc:ccc} \mathbf{M}_1 & \mathbf{M}_2 & \mathbf{M}_3 & \mathbf{N}_1 & \mathbf{N}_2 & \mathbf{N}_3 \\ \mathbf{M}_4 & \mathbf{M}_5 & \mathbf{M}_6 & \mathbf{N}_4 & \mathbf{N}_5 & \mathbf{N}_6 \\ \mathbf{M}_7 & \mathbf{M}_8 & \mathbf{M}_9 & \mathbf{N}_7 & \mathbf{N}_8 & \mathbf{N}_9 \\ \hdashline \mathbf{Q}_1 & \mathbf{Q}_2 & \mathbf{Q}_3 & \mathbf{R}_1 & \mathbf{R}_2 & \mathbf{R}_3 \\ \mathbf{Q}_4 & \mathbf{Q}_5 & \mathbf{Q}_6 & \mathbf{R}_4 & \mathbf{R}_5 & \mathbf{R}_6 \\ \mathbf{Q}_7 & \mathbf{Q}_8 & \mathbf{Q}_9 & \mathbf{R}_7 & \mathbf{R}_8 & \mathbf{R}_9 \end{array} \right)^{-1} \quad (32)
$$

Then when all consist of the same level shifts or the upper level shifts (suppose $\mathbf{X} > \mathbf{Y} > \mathbf{Z}$),

$\boldsymbol{K}_4, \boldsymbol{K}_7, \boldsymbol{K}_8, \boldsymbol{L}_4, \boldsymbol{L}_7, \boldsymbol{L}_8, \boldsymbol{M}_2, \boldsymbol{M}_3, \boldsymbol{M}_6, \boldsymbol{N}_4, \boldsymbol{N}_7, \boldsymbol{N}_8, \boldsymbol{R}_2, \boldsymbol{R}_3, \boldsymbol{R}_4$

are all 0.

As

$\boldsymbol{M}_4 = \boldsymbol{M}_2^T,\ \boldsymbol{M}_7 = \boldsymbol{M}_3^T,\ \boldsymbol{M}_8 = \boldsymbol{M}_6^T,\ \boldsymbol{R}_6 = \boldsymbol{R}_2^T,\ \boldsymbol{R}_7 = \boldsymbol{R}_3^T,\ \boldsymbol{R}_8 = \boldsymbol{R}_6^T,\ \boldsymbol{Q}_2 = \boldsymbol{N}_4^T,\ \boldsymbol{Q}_3 = \boldsymbol{N}_7^T,\ \boldsymbol{Q}_6 = \boldsymbol{N}_8^T,$

therefore they are all 0.

$\boldsymbol{M}_1, \boldsymbol{M}_5, \boldsymbol{M}_9, \boldsymbol{R}_1, \boldsymbol{R}_5, \boldsymbol{R}_9$ become diagonal Matrices.

By using a symbol "$*$" as a diagonal matrix, $\mathbf{P}$ becomes as follows by using the relation stated above.

$$\mathbf{P} = \left(\begin{array}{ccc:ccc} \mathbf{K}_1, & \mathbf{K}_2, & \mathbf{K}_3, & \mathbf{L}_1, & \mathbf{L}_2, & \mathbf{L}_3 \\ \mathbf{0}, & \mathbf{K}_5, & \mathbf{K}_6, & \mathbf{0}, & \mathbf{L}_5, & \mathbf{L}_6 \\ \mathbf{0}, & \mathbf{0}, & \mathbf{K}_9, & \mathbf{0}, & \mathbf{0}, & \mathbf{L}_9 \end{array}\right)$$
$$\times \left(\begin{array}{ccc:ccc} *, & \mathbf{0}, & \mathbf{0}, & \mathbf{N}_1, & \mathbf{N}_2, & \mathbf{N}_3 \\ \mathbf{0}, & *, & \mathbf{0}, & \mathbf{0}, & \mathbf{N}_5, & \mathbf{N}_6 \\ \mathbf{0}, & \mathbf{0}, & *, & \mathbf{0}, & \mathbf{0}, & \mathbf{N}_9 \\ \hdashline \mathbf{Q}_1, & \mathbf{0} & \mathbf{0}, & *, & \mathbf{0}, & \mathbf{0} \\ \mathbf{Q}_4, & \mathbf{Q}_5, & \mathbf{0}, & \mathbf{0}, & *, & \mathbf{0} \\ \mathbf{Q}_7, & \mathbf{Q}_8, & \mathbf{Q}_9, & \mathbf{0}, & \mathbf{0}, & * \end{array}\right)^{-1} \quad (33)$$

5. NUMERICAL EXAMPLE

We consider the case that the brand selection shifts to the same class or upper classes. As above-referenced, the transition matrix must be an upper triangular matrix.

Suppose the following events occur.

	X_{t-2} to X_{t-1}			X_{t-1} to X_t		
①	L_1	L_1	2 *events*	L_1	L_1	2 *events*
②	L_1	L_2	1 *event*	L_2	L_2	1 *event*
③	L_2	L_2	3 *events*	L_2	L_3	3 *events*
④	L_3	L_3	1 *event*	L_3	L_3	1 *event*
⑤	L_2	L_2	2 *events*	L_2	L_2	2 *events*
⑥	L_1	L_1	1 *event*	L_1	M_1	1 *event*
⑦	L_1	L_1	2 *events*	L_1	M_2	2 *events*
⑧	L_1	L_1	3 *events*	L_1	M_3	3 *events*
⑨	L_2	L_2	1 *event*	L_2	M_1	1 *event*
⑩	L_2	L_2	1 *event*	L_2	M_2	1 *event*
⑪	L_2	L_2	2 *events*	L_2	M_3	2 *events*
⑫	L_3	L_3	1 *event*	L_3	M_1	1 *event*
⑬	L_3	L_3	3 *events*	L_3	M_2	3 *events*
⑭	L_3	L_3	2 *events*	L_3	M_3	2 *events*
⑮	L_1	L_2	1 *event*	L_2	M_1	1 *event*
⑯	L_1	L_2	1 *event*	L_2	M_2	1 *event*
⑰	L_1	L_3	1 *event*	L_3	M_3	1 *event*
⑱	L_2	L_3	2 *events*	L_3	M_1	2 *events*
⑲	L_2	L_3	2 *events*	L_3	M_2	2 *events*
⑳	L_2	L_3	1 *event*	L_3	M_3	1 *event*
㉑	M_1	M_1	3 *events*	M_1	M_1	3 *events*
㉒	M_1	M_2	3 *events*	M_2	M_2	3 *events*

	X_{t-2} to X_{t-1}			X_{t-1} to X_t		
㉓	M_1	M_2	2 events	M_2	M_3	2 events
㉔	M_2	M_2	1 event	M_2	M_2	1 event
㉕	M_2	M_3	1 event	M_3	M_3	1 event
㉖	M_3	M_3	2 events	M_3	M_3	2 events
㉗	M_1	M_1	2 events	M_1	U_1	2 events
㉘	M_1	M_1	1 event	M_1	U_2	1 event
㉙	M_1	M_1	3 events	M_1	U_3	3 events
㉚	M_1	M_2	2 events	M_2	U_1	2 events
㉛	M_1	M_2	1 event	M_2	U_2	1 event
㉜	M_1	M_2	2 events	M_2	U_3	2 events
㉝	M_1	M_3	1 event	M_3	U_1	1 event
㉞	M_1	M_3	1 event	M_3	U_2	1 event
㉟	M_2	M_2	2 events	M_2	U_1	2 events
㊱	M_2	M_2	2 events	M_2	U_2	2 events
㊲	M_2	M_2	3 events	M_2	U_3	3 events
㊳	M_2	M_3	2 events	M_3	U_1	2 events
㊴	M_2	M_3	1 event	M_3	U_2	1 event
㊵	M_2	M_3	2 events	M_3	U_3	2 events
㊶	M_3	M_3	1 event	M_3	U_1	1 event
㊷	M_3	M_3	2 events	M_3	U_2	2 events
㊸	M_3	M_3	2 events	M_3	U_3	2 events
㊹	U_1	U_1	1 event	U_1	U_1	1 event
㊺	U_1	U_1	1 event	U_1	U_2	1 event
㊻	U_1	U_1	2 events	U_1	U_3	2 events
㊼	U_2	U_2	1 event	U_2	U_2	1 event
㊽	U_2	U_2	2 events	U_2	U_3	2 events
㊾	U_2	U_2	2 events	U_2	U_1	2 events
㊿	U_3	U_3	3 events	U_3	U_3	3 events
51	U_3	U_3	2 events	U_3	U_2	2 events
52	U_3	U_3	1 event	U_3	U_1	1 event
53	M_3	M_3	1 event	M_3	M_2	1 event
54	M_3	M_3	2 events	M_3	M_1	2 events
55	M_3	M_2	1 event	M_2	M_2	1 event
56	M_3	M_2	1 event	M_2	M_1	1 event
57	M_2	M_2	2 events	M_2	M_1	2 events
58	L_3	L_3	3 events	L_3	L_2	3 events
59	L_3	L_2	2 events	L_2	L_1	2 events
60	L_3	L_1	1 event	L_1	L_1	1 event
61				L_1	L_1	2 events
62				L_1	L_2	2 events
63				L_1	L_3	1 event

	X_{t-2} to X_{t-1}	X_{t-1} to X_t		
64		L_1	M_1	1 *event*
65		L_1	M_2	3 *events*
66		L_2	L_2	2 *events*
67		L_2	L_3	1 *event*
68		L_2	L_1	2 *events*
69		L_2	M_1	1 *event*
70		L_2	U_1	1 *event*
71		L_1	U_2	3 *events*
72		M_1	M_1	2 *events*
73		L_3	L_3	2 *events*
74		M_2	M_2	1 *event*
75		M_3	M_3	1 *event*
76		M_1	M_2	3 *events*
77		M_1	U_1	3 *events*
78		M_2	U_3	2 *events*
79		M_3	U_2	1 *event*
80		U_1	U_1	1 *event*
81		U_2	U_2	2 *events*
82		U_3	U_3	2 *events*
83		U_1	U_2	1 *event*
84		U_1	U_3	2 *events*
85		U_3	U_2	2 *events*
86		U_3	U_1	1 *event*
87		U_2	U_1	1 *event*
88		U_2	U_3	2 *events*
89		L_3	L_1	3 *events*
90		M_2	L_1	1 *event*

Vector $\begin{pmatrix}\boldsymbol{X}_{t-2}\\ \boldsymbol{Y}_{t-2}\\ \boldsymbol{Z}_{t-2}\end{pmatrix}, \begin{pmatrix}\boldsymbol{X}_{t-1}\\ \boldsymbol{Y}_{t-1}\\ \boldsymbol{Z}_{t-1}\end{pmatrix}, \begin{pmatrix}\boldsymbol{X}_{t}\\ \boldsymbol{Y}_{t}\\ \boldsymbol{Z}_{t}\end{pmatrix}$ in these cases are expressed as follows.

We show ① and ② cases for example.

$$① \quad \begin{pmatrix}\mathbf{X}_{t-2}\\ \\ \mathbf{Y}_{t-2}\\ \\ \mathbf{Z}_{t-2}\end{pmatrix} = \begin{array}{c} X_{t-2}\\ \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix}\end{array}, \quad \begin{pmatrix}\mathbf{X}_{t-1}\\ \\ \mathbf{Y}_{t-1}\\ \\ \mathbf{Z}_{t-1}\end{pmatrix} = \begin{array}{c} X_{t-1}\\ \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix}\end{array}, \quad \begin{pmatrix}\mathbf{X}_{t}\\ \\ \mathbf{Y}_{t}\\ \\ \mathbf{Z}_{t}\end{pmatrix} = \begin{array}{c} X_{t}\\ \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\\0\\0\end{pmatrix}\end{array}$$

② $$\begin{pmatrix} \mathbf{X}_{t-2} \\ \\ \mathbf{Y}_{t-2} \\ \\ \mathbf{Z}_{t-2} \end{pmatrix} = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\1\\0\\0 \end{pmatrix}, \quad \begin{pmatrix} \mathbf{X}_{t-1} \\ \\ \mathbf{Y}_{t-1} \\ \\ \mathbf{Z}_{t-1} \end{pmatrix} = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\1\\0 \end{pmatrix}, \quad \begin{pmatrix} \mathbf{X}_{t} \\ \\ \mathbf{Y}_{t} \\ \\ \mathbf{Z}_{t} \end{pmatrix} = \begin{pmatrix} 0\\0\\0\\0\\0\\0\\0\\1\\0 \end{pmatrix}$$

Substituting these to equation (30), we obtain the following estimated Matrix.

$$\hat{\mathbf{P}} = \begin{pmatrix}
2 & 3 & 2 & 5 & 8 & 6 & 0 & 1 & 0 & 1 & 2 & 1 & 5 & 4 & 1 & 0 & 0 & 0 \\
2 & 3 & 4 & 1 & 4 & 6 & 0 & 3 & 0 & 1 & 1 & 2 & 3 & 3 & 2 & 0 & 0 & 0 \\
4 & 4 & 5 & 3 & 7 & 4 & 0 & 0 & 0 & 2 & 2 & 3 & 5 & 5 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 5 & 0 & 0 & 2 & 3 & 3 & 0 & 0 & 0 & 3 & 0 & 3 & 2 & 3 & 1 \\
0 & 0 & 0 & 3 & 5 & 0 & 5 & 2 & 5 & 0 & 0 & 0 & 3 & 1 & 2 & 3 & 3 & 3 \\
0 & 0 & 0 & 0 & 2 & 4 & 3 & 2 & 4 & 0 & 0 & 0 & 2 & 1 & 2 & 4 & 3 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 5 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 2 & 5 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1
\end{pmatrix}$$

$$\times \begin{pmatrix}
8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 26 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10 & 10 & 2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 20 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 6 & 10 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 9 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 5 & 10 \\
4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 9 & 10 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 21 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 10 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 8 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 12 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 13
\end{pmatrix}^{-1}$$

$$
= \begin{pmatrix}
0.250 & 0.200 & 0.200 & 0.522 & 0.751 & 0.912 & 0.016 & 0.087 & 0.040 & 0 & 0.200 & -0.033 & -0.430 & -0.561 & -0.802 & -0.036 & -0.070 & -0.045 \\
0.250 & 0.400 & 0.400 & 0.104 & 0.282 & 0.521 & 0.047 & 0.262 & 0.120 & 0 & -0.200 & -0.067 & -0.085 & -0.184 & -0.314 & -0.107 & -0.211 & -0.136 \\
0.500 & 0.400 & 0.400 & 0.130 & 0.174 & 0.130 & 0 & 0 & 0 & 0 & 0 & 0.100 & 0.087 & 0.155 & 0.029 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.151 & -0.304 & -0.497 & 0.062 & 0.086 & 0.132 & 0 & 0 & 0 & 0.270 & 0.376 & 0.715 & 0.093 & 0.112 & -0.042 \\
0 & 0 & 0 & 0.184 & 0.186 & -0.107 & 0.191 & -0.001 & 0.143 & 0 & 0 & 0 & -0.014 & -0.013 & 0.225 & 0.111 & 0.164 & 0.106 \\
0 & 0 & 0 & -0.007 & 0.125 & 0.277 & 0.009 & -0.063 & 0.066 & 0 & 0 & 0 & 0.012 & -0.119 & -0.085 & 0.337 & 0.231 & 0.112 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.289 & 0.340 & 0.231 & 0 & 0 & 0 & 0 & 0 & 0 & -0.130 & -0.301 & -0.021 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.120 & 0.275 & 0.126 & 0 & 0 & 0 & 0 & 0 & 0 & -0.076 & -0.079 & 0.082 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.111 & 0.228 & 0.215 & 0 & 0 & 0 & 0 & 0 & 0 & -0.149 & -0.009 & -0.132
\end{pmatrix}
$$

The Block Matrices make upper triangular matrix as was supposed. We can confirm that $\boldsymbol{K}_4, \boldsymbol{K}_7, \boldsymbol{K}_8, \boldsymbol{L}_4, \boldsymbol{L}_7, \boldsymbol{L}_8, \boldsymbol{M}_2, \boldsymbol{M}_3, \boldsymbol{M}_4, \boldsymbol{M}_6, \boldsymbol{M}_7, \boldsymbol{M}_8, \boldsymbol{N}_4, \boldsymbol{N}_7, \boldsymbol{N}_8, \boldsymbol{Q}_2, \boldsymbol{Q}_3, \boldsymbol{Q}_6, \boldsymbol{R}_2, \boldsymbol{R}_3, \boldsymbol{R}_4, \boldsymbol{R}_6, \boldsymbol{R}_7, \boldsymbol{R}_8$ are all 0. $\boldsymbol{M}_1, \boldsymbol{M}_5, \boldsymbol{M}_9, \boldsymbol{R}_1, \boldsymbol{R}_5, \boldsymbol{R}_9$ become diagonal Matrices as we have assumed.

6. APPLICATION OF THIS METHOD

Consumers' behavior may converge by repeating forecast with above method and the total sales of all brands may be reduced. Therefore, the analysis results suggest when and what to put new brand into the market which contribute to the expansion of the market.

An application of this method can be considered in the following case. There may arise following case. Consumers and producers do not recognize the brand position clearly. But the analysis of consumers' behavior let them know their brand position in the market. In such a case, strategic marketing guidance to select brand would be introduced.

Setting in order the brand position of various goods and taking suitable marketing policy, enhancement of sales would be enabled. Setting higher ranked brand, consumption would be promoted.

7. CONCLUSION

Consumers often buy products of a higher grade brand as they are accustomed or bored with their current brand.

In this paper, the block matrix structure under brand groups was clarified when the brand selection was executed toward higher grade brand. And formulation of expansion to the second order lag was conducted using Block Matrix. An s-step forecast model was also formulated. In the numerical example, matrix structure's hypothesis was verified. Such research as questionnaire investigation of consumers activity in automobile purchasing should be executed in the near future to verify obtained results. This method would be applicable in such fields as brand bag purchasing, brand wine purchasing, brand dress purchasing etc. This new method should be examined in various cases.

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