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A Hybrid Method to Improve Forecasting Accuracy With An Application to Stock Price Data

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Abstract. *In industries, how to improve forecasting accuracy in areas such as sales, shipping and stock price is an important issue. Noting that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily, but in this paper, we utilize the above stated theoretical solution. Combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following use of this method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original stock price data. The weights for these functions are set at 0.5 for two patterns at first and then varied by a 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. Comparison between this method and non trend removing method is executed. Good result is obtained.*

Key Words*: minimum variance, exponential smoothing method, forecasting, trend*

1. INTRODUCTION

High accuracy forecasting is needed for suitable decision making. If it is not done properly, it misleads decision making.

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM)[1]−[4]. Among these, ESM is said to be a practical, simple method.

For this method, various improvements such as adding compensating item for time lag, coping with the time series with trend [5], utilizing Kalman Filter [6], Bayes Forecasting [7], adaptive ESM [8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda [6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise. It can be said that it does not pursue an optimum solution from the very data themselves which should be derived by those estimations. Ishii [11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show an analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before [12] [13]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing the method stated above, a revised forecasting method is proposed. In making a forecast such as sales data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original stock price data of electronics companies. The weights for these functions are set at 0.5 for two patterns at first and then varied by 0.01 increments for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. In this research, stock price data of three electronics companies were used as an example. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. Comparison between this method and non trend removing method is executed. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred to in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2. DESCRIPTION OF ESM USING ARMA MODEL

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha\left(x_t - \hat{x}_t\right) \tag{1}$$

$$= \alpha x_t + \left(1-\alpha\right)\hat{x}_t \tag{2}$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$

x_t : realized value at t

α : smoothing constant $\left(0 < \alpha < 1\right)$

(2) is re-stated as :

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha\left(1-\alpha\right)^l x_{t-l} \tag{3}$$

By the way, we consider the following (1,1) order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \tag{4}$$

Generally, (p,q) order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{5}$$

Here,

$\{x_t\}$: Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (5) is supposed to satisfy convertibility condition. Utilizing the relation that

$$E\left[e_t \middle| e_{t-1}, e_{t-2}, \cdots\right] = 0$$

we get the following equation from (4).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \tag{6}$$

Operating this scheme on $t+1$, we finally get

$$\begin{aligned} \hat{x}_{t+1} &= \hat{x}_t + \left(1-\beta\right)e_t \\ &= \hat{x}_t + \left(1-\beta\right)\left(x_t - \hat{x}_t\right) \end{aligned} \tag{7}$$

If we set $1-\beta = \alpha$, the above equation is the same as (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1.

Comparing with (4) and (5), we obtain

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases}$$

From (1), (7),

$$\alpha = 1-\beta$$

Therefore, we get

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \tag{8}$$

From the above, we can get an estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (5) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \tag{9}$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \tag{10}$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (9), (10), we get the following non-linear equations which are well known.

$$\left.\begin{array}{l} \tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{array}\right\} \tag{11}$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (4) (5) (8) (11), we get

$$\left.\begin{array}{l} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \\ \tilde{r}_0 = \left(1 + b_1^2\right)\sigma_e^2 \\ \tilde{r}_1 = b_1 \sigma_e^2 \end{array}\right\} \tag{12}$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \tag{13}$$

the following equation is derived.

$$\rho_1 = \frac{b_1}{1+b_1^2} \tag{14}$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1-4\rho_1^2}}{2\rho_1} \tag{15}$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2} \tag{16}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (14), using the next relation,

$$(1-b_1)^2 \ge 0$$
$$(1+b_1)^2 \ge 0$$

(16) always holds.
As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0$$

Finally we get

$$\left.\begin{aligned} b_1 &= \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \tag{17}$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way. Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter. It can be estimated only by calculating 0th and 1st order autocorrelation function.

3. TREND REMOVAL METHOD

As trend removal method, we describe the combination of linear and non-linear function.
[1] Linear function
We set

$$y = a_1 x + b_1 \tag{18}$$

as a linear function.

[2] Non-linear function
We set

$$y = a_2 x^2 + b_2 x + c_2 \tag{19}$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \tag{20}$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function.
We set

$$y = \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2) \tag{21}$$

$$y = \beta_1 (a_1 x + b_1) + \beta_2 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \tag{22}$$

$$\begin{aligned} y = {} & \gamma_1 (a_1 x + b_1) + \gamma_2 (a_2 x^2 + b_2 x + c_2) \\ & + \gamma_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \end{aligned} \tag{23}$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Here, $\alpha_2 = 1 - \alpha_1$, $\beta_2 = 1 - \beta_1$,. $\gamma_3 = 1 - (\gamma_1 + \gamma_2)$. Comparative discussion concerning (21), (22) and (23) are described in section 5.

4. MONTHLY RATIO

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\}\ (i = 1, \cdots, L)\ (j = 1, \cdots, 12)$$

Where, $x_{ij} \in R$ in which j means month and i means year and x_{ij} is a stock price data of i-th year, j-th month. Then, monthly ratio $\tilde{x}_j$ $(j = 1, \cdots, 12)$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L}\sum_{i=1}^{L} x_{ij}}{\frac{1}{L}\cdot\frac{1}{12}\sum_{i=1}^{L}\sum_{j=1}^{12} x_{ij}} \tag{24}$$

Monthly trend is removed by dividing the data by (24). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

5. FORECASTING THE STOCK PRICE DATA OF ELECTRONICS COMPANIES

5.1 Analysis Procedure

The original stock price data of three electronics companies from December 2008 to November 2011 are analyzed. Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data (1 to 24) and remove trend by the method stated in 3. Then we calculate monthly ratio by the method stated in 4. After removing monthly trend, the method stated in 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{25}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N} \varepsilon_i \tag{26}$$

Variance of forecasting error is calculated by:

$$\sigma_\varepsilon^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{27}$$

5.2 Trend Removing

Trend is removed by dividing original data by, (21), (22), (23). The patterns of trend removal are exhibited in Table1.

Table 1: The patterns of trend removal

Pattern1	α_1, α_2 are set 0.5 in the equation (21)
Pattern2	β_1, β_2 are set 0.5 in the equation (22)
Pattern3	α_1 is shifted by 0.01 increment in (21)
Pattern4	β_1 is shifted by 0.01 increment in (22)
Pattern5	γ_1 and γ_2 are shifted by 0.01 increment in (23)

In pattern1 and 2, the weight of α_1, α_2, β_1, β_2 are set 0.5 in the equation (21), (22). In pattern 3, the weight of α_1 is shifted by 0.01 increment in (21) which satisfies the range $0 \le \alpha_1 \le 1.00$. In pattern 4, the weight of β_1 is shifted in the same way which satisfies the range $0 \le \beta_1 \le 1.00$. In pattern 5, the weight of γ_1 and γ_2 are shifted by 0.01 increment in (23) which satisfies the range $0 \le \gamma_1 \le 1.00$, $0 \le \gamma_2 \le 1.00$. The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (18), (19) and (20) are exhibited in Table 2. Estimation results of weights of (21), (22) and (23) are exhibited in table 3.

Table 2: Coefficient of (18),(19) and (20)

	1st		2nd			3rd			
	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
Panasonic Co.	-3.315	1393.018	-2.138	50.131	1161.419	0.100	-5.888	88.406	1073.657
Ricoh Co.	2.455	1311.815	-1.443	38.539	1155.452	-0.082	1.637	7.105	1227.526
Konica Minolta Holdings, Inc.	4.861	914.239	-1.707	47.527	729.353	-0.024	-0.822	38.499	750.053

Table 3: weights of (21), (22) and (23)

	monthly	pattern 1		pattern 2		pattern 3		pattern 4		pattern 5		
	ratio	α_1	α_2	β_1	β_2	α_1	α_2	β_1	β_2	γ_1	γ_2	γ_3
Panasonic Co.	used	0.5	0.5	0.5	0.5	0.71	0.29	0.24	0.76	0.02	0.15	0.83
	not used	0.5	0.5	0.5	0.5	0.89	0.11	1	0	0.89	0.11	0
Ricoh Co.	used	0.5	0.5	0.5	0.5	1	0	0.4	0.6	0	0.35	0.65
	not used	0.5	0.5	0.5	0.5	1	0	0.66	0.34	0.66	0	0.34

Konica Minolta	used	0.5	0.5	0.5	0.5	0	1	0.53	0.47	0	0.53	0.47
Holdings, Inc.	not used	0.5	0.5	0.5	0.5	0	1	0.6	0.4	0	0.74	0.26

5.3 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing trend, monthly ratio is calculated by the method stated in 4. Next, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (17). There are cases that we cannot obtain a theoretical solution for because they do not satisfy the condition of (16). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 4.

Table 4: Estimated Smoothing Constant with Minimum Variance

	monthly ratio	pattern 1		pattern 2		pattern 3		pattern 4		pattern 5	
		ρ_1	α	ρ_1	α	ρ_1	α	ρ_1	α	ρ_1	α
Panasonic Co.	used	-0.1187	0.8795	-0.0832	0.9162	-0.1253	0.8726	-0.0544	0.9454	-0.0377	0.9622
	not used	-0.2285	0.7581	-0.2315	0.7546	-0.2294	0.7571	-0.2295	0.7569	-0.2294	0.7571
Ricoh Co.	used	-0.0873	0.9120	-0.1166	0.8818	-0.0748	0.9248	-0.1188	0.8794	-0.0935	0.9056
	not used	-0.0920	0.9072	-0.0939	0.9053	-0.0641	0.9357	-0.0863	0.9131	-0.0863	0.9131
Konica Minolta Holdings, Inc.	used	-0.0755	0.9241	-0.0763	0.9233	-0.0637	0.9361	-0.0754	0.9242	-0.0649	0.9349
	not used	-0.4116	0.4750	-0.4124	0.4730	-0.4123	0.4733	-0.4121	0.4737	-0.4126	0.4726

5.4 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (27). Forecasting results are exhibited in Figure 1, 2, 3 for the cases that monthly ratio is not used.

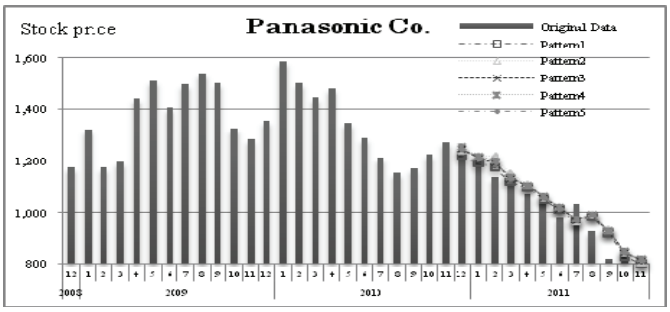

Figure 1: Forecasting Results of PANASONIC CO.

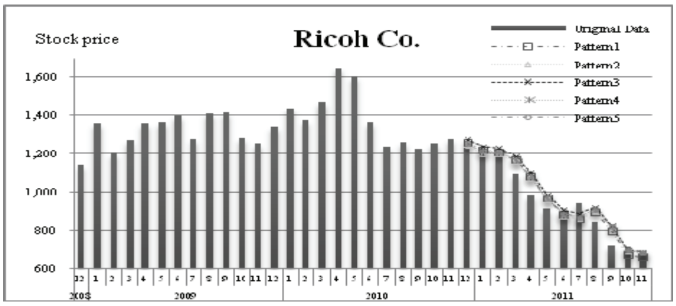

Figure 2: Forecasting Results of RICOH CO.

Variance of forecasting error is exhibited in Table 5.

Figure 3: Forecasting Results of KONICA MINOLTA HOLDINGS, INC.

Table 5: Variance of Forecasting Error

	monthly ratio	pattern 1	pattern 2	pattern 3	pattern 4	pattern 5
Panasonic CO.	used	3398.240	3214.069	3298.166	3075.034	3060.747
	not used	1584.671	1864.072	1537.780	1540.675	1537.780
Ricoh CO.	used	8902.690	7436.394	8633.977	7402.685	7328.860
	not used	3231.870	3009.797	3069.847	2991.417	2991.417
Konica Minolta Holdings, Inc.	used	4268.878	2631.804	3756.030	2462.500	2454.436
	not used	1812.227	1691.377	1732.147	1669.923	1610.550

Here, we make comparison of this newly proposed method and the non trend removing method. The non trend removing method utilizes only the theoretical solu tion of smoothing constant in ESM.

Table 6: Variance of Forecasting Error

	Theoretical solution	the revised forecasting method
Panasonic CO.	15103.212	1537.780
Ricoh CO.	16381.935	2991.417
Konica Minolta Holdings, Inc.	6888.313	1610.550

5.5 Remarks

In all cases, that monthly trend is not removed had a better forecasting accuracy. The result of Table 6 shows that this revised forecasting method had by far a better forecasting accuracy than those of a mere application of theoretical solution of smoothing constant in ESM. The stock price data of Panasonic Corporation had a good result in pattern 3 ($1^{st}+2^{nd}$ order), Ricoh Corporation in pattern 4 ($1^{st}+3^{rd}$ order) and pattern 5 (1st+2nd+3rd order), and Konica Minolta Holdings, Incorporated in and pattern 5 ($1^{st}+2^{nd}+3^{rd}$ order).

6. CONCLUSION

In this paper, we utilized a theoretical solution in ESM. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the original stock price data of electronics companies. The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. The calculation results show that the method where optimal weights are searched by 0.01 increment of pattern 5 is best in forecasting accuracy. Comparison between this method and non trend removing method is executed. The stock price data of Panasonic Corporation had a good result in pattern 3 ($1^{st}+2^{nd}$ order), Ricoh Corporation in pattern 4 ($1^{st}+3^{rd}$ order) and pattern 5 ($1^{st}+2^{nd}+3^{rd}$ order), and Konica Minolta Holdings, Incorporated in and pattern 5 ($1^{st}+2^{nd}+3^{rd}$ order). The result showed that this revised forecasting method had by far a better forecasting accuracy than those of a mere application of theoretical solution of smoothing constant in ESM. Various cases should be examined hereafter.

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